

Lattices and the Geometry of Numbers

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1. ABSTRACT

In this paper we discuss about properties of lattices and its application in theoretical and algorithmic number theory. This result of Minkowski regarding the lattices initiated the subject of Geometry of Numbers, which uses geometry to study the properties of algebraic numbers. It has application on various other fields of mathematics especially the study of Diophantine equations, analysis of functional analysis etc. This paper gives an elementary introduction to the field of geometry of numbers. In this paper we shall first give a broad overview of the concept of lattice and then discuss about the geometrical properties it has and its applications.

2. LATTICE

Before introducing Minkowski's theorem we shall first discuss what is a lattice.

Definition 1: A lattice τ is a subgroup of \mathbb{R}^n such that it can be represented as

$$\tau = \alpha_1 \mathbb{Z} + \alpha_2 \mathbb{Z} + \dots + \alpha_m \mathbb{Z}$$

Here $\{\alpha_i\}$ are linearly independent vectors of the space \mathbb{R}^n and $m \leq n$. Here \mathbb{Z} is the set of whole numbers.

We call these vectors $\{\alpha_i\}$ the basis of the lattice. By the definition we can see that a lattice is a subgroup and a free abelian group of rank m , of the vector space \mathbb{R}^n . The rank and dimension of the lattice is m and n respectively, the lattice will be complete if $m = n$. This definition is not only limited to the vector space \mathbb{R}^n . It can be extended to any arbitrary field F , in which the basis vectors $\{\alpha_i\}$ will belong to the field F . In this article we shall discuss about both complete or full-rank lattices and incomplete lattices. We will now define another terms known as the fundamental mesh.

Definition 2: The set of elements which can be denoted as

$$\varphi(\alpha) = \{\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_m v_m | v_i \in \mathbb{R}, 0 \leq v_i \leq 1\}$$

is called a fundamental mesh of the lattice.

A very important thing to notice is that not every given set of vectors $\{\alpha_i\}$ forms the basis of a given lattice τ . In the next lemma we shall state the condition for a given set of vectors $\{\alpha_i\}$ to form the basis of a given lattice τ .

Lemma 1: A given set of vectors $\{\alpha_i\}$ form the basis of a given lattice τ if it satisfies the following condition:

$$\varphi(\alpha) \cap \tau = \{\mathbf{0}\}$$

Proof: The lattice τ is the set of all their integer combinations of the basis vectors $\{\alpha_i\}$. We also know that $\varphi(\alpha)$ is the set of linear combinations of basis vectors $\{\alpha_i\}$ with the coefficients $v_i \in \mathbb{R}, 0 \leq v_i \leq 1$. Therefore the only element in common is the zero vector $\mathbf{0}$.

Let us now state a lemma regarding the properties of the lattice τ . This proof can be found in many articles and books. We state here the elegant proof as stated and given by Comcaux¹ and Neukirch²

Geometry Of Numbers

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